An Analytical Evaluation of the Factor k^2 for Protective Conductors

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***Abstract*—At the occurrence of phase-to-ground faults, abnormal levels of thermal energy I^2t, due to the Joule effect, will be developed during the clearing time that protective devices take to operate. The I^2t, also referred to as specific energy or Joule Integral, is accumulated within the elements forming the fault loop, such as the protective conductors (also referred to as equipment grounding conductors), responsible to return ground-fault currents to the source. As a consequence, the temperature of these conductors elevates and may exceed, in the case of an incorrect design, the maximum value that their insulation can withstand. This dangerous situation can cause the failure of the conductor insulation and/or trigger fires in neighboring materials. The maximum I^2t that protective conductors can endure is, therefore, crucial in order to guarantee the electrical safety. The parameters on which the maximum I^2t depends are described by the factor k^2, which will be herein discussed and analytically evaluated. The intention of the authors is to provide a theoretical support to the Power Systems Grounding Working Group of the Technical Books Coordinating Committee IEEE P3003.2 "Recommended Practice for Equipment Grounding and Bonding in Industrial and Commercial Power Systems"; the working group is currently elaborating a dot standard based on IEEE Standard 142-2007, also referred to as the Green Book. To this purpose, a comparison with existing formulas, currently present in codes, standards of the International Electrotechnical Commission and of the IEEE, as well in the literature, will be also presented.**

***Index Terms*—Adiabatic, ampacity, cables, equipment grounding conductor (EGC), fault duration, ground faults, I^2t, Joule integral, protective conductor, protective device.**

Nomenclature

$i_G(t)$	Instantaneous ground-fault current.
θ_0	Initial temperature of the protective conductor.
θ_f	Final temperature of the protective conductor.
θ_M	Maximum temperature that protective conductor insulation can withstand without damage.
R	Fault-loop resistance.
EGC	Equipment grounding conductor.
PE	Protective conductor.
ρ_0	Resistivity at 0 °C.
ρ_{20}.	Resistivity at 20 °C.

I. Introduction

THIS PAPER seeks to provide a theoretical validation of existing formulas for sizing *protective conductors* (PE) (also referred to as *equipment grounding conductors*, EGCs) currently in use in codes, standards of the International Electrotechnical Commission (IEC) and IEEE standards, such as, for example, [1] and [2].

Properly sizing PEs is extremely important, as at the occurrence of phase-to-ground faults, abnormal levels of thermal energy $I_G^2 t$, also referred to as Joule Integral, occur. This energy develops during the clearing time that protective devices take to operate and disconnect the faulty equipment. Such let-through energy needs to be compared with the maximum thermal energy that a given protective conductor can endure without damaging. The evaluation of protective conductors' maximum thermal energy is, therefore, crucial in order to guarantee the electrical safety of persons under ground-fault conditions.

This maximum admissible thermal energy of the PE not only depends on its cross-sectional area, but also on its constituting material (e.g., copper), its type of insulation (e.g., PVC), its initial temperature θ_0 at the inception of the fault, and the maximum temperature θ_M that the conductor insulation can withstand without damage.

The initial temperature θ_0 may be taken as the conductor maximum operating temperature in correspondence with its current-carrying capability. This conservative assumption, which may result in protective conductors oversizing, is indeed more appropriate for line conductors involved in short circuits. In these cases, in fact, the initial temperature of the conductors at the inception of the short is the actual temperature in correspondence with the prefault load current; such temperature is conservatively assumed as that in correspondence with the ampacity[1] of the cable.

On the contrary, the PE is generally "at rest," as no current normally circulates through it. Thus, if the protective conductor is not incorporated in cables, and not bunched with other cables, its initial temperature may be the ambient temperature (conventionally 30 °C).

[1]Ampacity is defined as the maximum amount of electrical current a conductor can carry while its insulation remains within its temperature rating. Exceeding temperature ratings shorten the useful life of conductors.

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II. K^2 Factor for Protective Conductors

To estimate the thermal stress to which protective conductors are subject, we can initially assume that the let-through energy is entirely accumulated within the PEs, and that there is no heat dissipation by convection or radiation by the conductor (i.e., adiabatic conditions).

As anticipated, in order for the protective conductor not to be damaged during the ground fault, the let-through energy must not exceed the maximum thermal energy that the PE can withstand. In formulas, and for the adiabatic case,

$$\int_0^{t_f} i_G^2 dt \leq k^2 S^2. \tag{1}$$

The left-hand side of (1) is the let-through energy developed during the fault [3]; i_G is the instantaneous ground-fault current, S the cross-sectional area of the protective conductor (mm^2), t_f is the clearing time of the protective device and k^2 is a factor that takes into account the resistivity, temperature coefficient and heat capacity of the conductor material, the initial temperature of the protective conductor at the inception of the fault, and the maximum admissible temperature the insulation of the PE can withstand without damage.

The factor k^2 is given by

$$k^2 = \frac{c}{\alpha_0 \cdot \rho_0} \ln \frac{1 + \alpha_0 \theta_M}{1 + \theta_0 \alpha_0}. \tag{2}$$

The Appendix provides an analytical calculation for the above factor k^2.

If in (2), we pose $\beta = 1/\alpha_0$ and $\rho_0 = \rho_{20^\circ}/(1 + 20^\circ \alpha_0)$, as per (A3), we obtain

$$k^2 = \frac{c(\beta + 20^\circ)}{\rho_{2_0}} \ln \left(1 + \frac{\theta_M - \theta_0}{\beta + \theta_0}\right) \tag{3}$$

which is an equivalent formulation of the k^2 factor that can be found in [4].

With the same positions, we can obtain the following formula, presented in [5]:

$$k^2 = \frac{c}{\rho_{20}} (20 + 1/\alpha_0) \ln \frac{(\theta_M + 1/\alpha_0)}{(\theta_0 + 1/\alpha_0)}. \tag{4}$$

Equations (3) and (4), included in different standards, do confirm the analytical calculation of the k^2 factor obtained in (2).

Equations (2)–(4) can be applied to conductor at different rated voltages, whose temperature limits, for various types of insulation, can be found in [5]–[8].

Equations (2)–(4) can also be applied under nonadiabatic conditions: differences in the calculated values of k^2 are only significant for smaller cross-sectional areas of cables (less than 10 mm^2).

TABLE I
Values of Parameters for Different Conductive Materials of PES

Conductive Material	Temperature coefficient of resistivity α_0 (°C^{-1})	Resistivity ρ_0 (Ω mm)	Volumetric heat capacity c (J/(°C mm^3))
Copper	$4.26 \cdot 10^{-3}$	$15.89 \cdot 10^{-6}$	$3.45 \cdot 10^{-3}$
Aluminum	$4.38 \cdot 10^{-3}$	$25.98 \cdot 10^{-6}$	$2.5 \cdot 10^{-3}$
Lead	$4.34 \cdot 10^{-3}$	$196.88 \cdot 10^{-6}$	$1.45 \cdot 10^{-3}$
Steel	$4.95 \cdot 10^{-3}$	$125.56 \cdot 10^{-6}$	$3.8 \cdot 10^{-3}$

TABLE II
Temperature Limits for Insulation Materials of Protective Conductors Not Incorporated in Cables, and Not Bunched With Other Cables

Conductor insulation	θ_0(°C)	θ_M(°C)
Paper	30	250
Polyvinyl chloride (PVC) (conductor cross-section ≤ 300 mm^2)	30	160
Polyvinyl chloride (PVC) (conductor cross-section > 300 mm^2)	30	140
Cross-linked polyethylene (XLPE)	30	250
60 °C Ethylene propylene rubber (EPR)	30	200
85 °C Ethylene propylene rubber (EPR)	30	220

III. Values of Parameters for Different Materials and Insulations of Protective Conductors

Table I lists values of parameters for different conductive materials of PEs to be used in the calculation of k^2:

The nature of adjacent insulating materials limits the maximum admissible temperatures of protective conductors.

In the following tables, temperature limits for protective conductors for ground-fault durations not exceeding 5 s are listed. If ground-fault clearing times exceed 5 s, maximum temperatures must be reduced according to the manufacturer's indications.

Temperature limits for insulation materials of protective conductors not incorporated in cables, and not bunched with other cables, are listed in Table II [4], [6].

Temperature limits for insulation materials of protective conductors as a core incorporated in a cable or not bunched with other cables or insulated conductors, are listed in Table III [4], [6].

Temperature limits for bare protective conductors in contact with cable covering, but not bunched with other cables, are listed in Table IV [4], [6].

Temperature limits for insulation materials of protective conductors as a metallic layer of a cable (e.g., armor, metallic sheath, concentric conductor, etc.) are listed in Table V [4], [6].

If protective conductors are bare and exposed to touch, or in contact with combustible materials, their superficial temperature may be a reason of concern. In normal condition areas, when there is no risk for the bare PE to cause damage to any neighboring material, the maximum temperature to consider for calculations should be 200 °C. However, different temperature limits can be adopted in different areas; if the bare protective conductor is well visible and confined in restricted zones, the maximum allowable temperature can be increased; on the other hand, if the bare PE is in fire risk locations, its maximum

TABLE III
TEMPERATURE LIMITS FOR INSULATION MATERIALS OF PROTECTIVE CONDUCTORS AS A CORE INCORPORATED IN A CABLE OR BUNCHED WITH OTHER CABLES OR INSULATED CONDUCTORS

I_G (A)	t_f (s)	$I_G^2 t$ (A²s)	$S_{30} \rightarrow S_{30\text{ trade size}}$ (mm²)	$S_{90} \rightarrow S_{90\text{ trade size}}$ (mm²)
1	2	3	4	5
20	991	$3.96 \cdot 10^5$	3.59→4	4.43→6
30	193	1.74105	2.37→2.5	2.94→4
40	53	$8.48 \cdot 10^4$	1.66→2.5	2.05→2.5
50	21.6	$5.40 \cdot 10^4$	1.32→1.5	1.64→2.5
60	20.8	$7.49 \cdot 10^4$	1.56→2.5	1.93→2.5
100	5.4	$5.40 \cdot 10^4$	1.32→1.5	1.64→2.5
200	1.73	$6.92 \cdot 10^4$	1.50→1.5	1.85→2.5
400	0.5	$8.00 \cdot 10^4$	1.61→2.5	1.99→2.5
500	0.37	$9.25 \cdot 10^4$	1.73→2.5	2.14→2.5
600	0.23	$8.28 \cdot 10^4$	1.64→2.5	2.03→2.5
700	0.18	$8.82 \cdot 10^4$	1.69→2.5	2.09→2.5
800	0.13	$8.32 \cdot 10^4$	1.64→2.5	2.03→2.5
1000	0.014	$1.40 \cdot 10^4$	0.67→1.5	0.83→1.5

TABLE IV
TEMPERATURE LIMITS FOR BARE PROTECTIVE CONDUCTORS IN CONTACT WITH CABLE COVERING, BUT NOT BUNCHED WITH OTHER CABLES

Cable covering	θ_0 (°C)	θ_M (°C)
Polyvinyl Chloride (PVC)	30	200
Cross-linked Polyethylene (XLPE)	30	150
Chlorosulphonated Polyethylene (CSP)	30	220

TABLE V
TEMPERATURE LIMITS FOR INSULATION MATERIALS OF PROTECTIVE CONDUCTORS AS A METALLIC LAYER OF A CABLE

Cable insulation	θ_0 (°C)	θ_M (°C)
70°C Polyvinyl chloride (PVC)	60	200
90°C Polyvinyl chloride (PVC)	80	200
Cross-linked polyethylene (XLPE)	80	200
60 °C Ethylene propylene rubber (EPR)	55	200
85 °C Ethylene propylene rubber (EPR)	75	220
Mineral PVC covered	70	200
Mineral bare sheath	105	250

TABLE VI
TEMPERATURE LIMITS FOR BARE PE WHERE THERE IS NO RISK OF DAMAGE TO ANY NEIGHBORING MATERIAL

Conditions of the bare PE	θ_0 (°C)	θ_M (°C)		
		Copper	Aluminum	Steel
Visible & in restricted areas	30	500	300	500
In normal conditions areas	30	200	200	200
In fire risk areas	30	150	150	150

temperature should be lowered. Table VI lists such temperature limits as per [4] and [6].

It should be noted that due to safety considerations, such as the risk of burns or of triggering fires or explosive atmospheres, the fusion temperatures of bare PEs, as maximum allowed temperatures, are not considered in the IEC world. Such temperatures would largely exceed the temperature limits of Table VI; in fact [1] indicates that if fusing is a criterion, then a final temperature of 1000 °C for copper and 630 °C for aluminum may be used.

TABLE VII
VALUES OF K^2 FOR INSULATED PROTECTIVE CONDUCTORS NOT INCORPORATED IN CABLES, AND NOT BUNCHED WITH OTHER CABLES

Conductor insulation	k^2 [A²s/mm²]		
	Copper	Aluminum	Steel
Paper	$3.08 \cdot 10^4$	$1.35 \cdot 10^4$	$4.08 \cdot 10^3$
Polyvinyl chloride (PVC) (conductor cross-section ≤ 300 mm²)	$2.04 \cdot 10^4$	$8.95 \cdot 10^3$	$2.72 \cdot 10^3$
Polyvinyl chloride (PVC) (conductor cross-section > 300 mm²)	$1.77 \cdot 10^4$	$7.79 \cdot 10^4$	$2.37 \cdot 10^4$
Cross-linked polyethylene (XLPE)	$3.08 \cdot 10^4$	$1.35 \cdot 10^4$	$4.08 \cdot 10^4$
60 °C Ethylene propylene rubber (EPR)	$2.53 \cdot 10^4$	$1.11 \cdot 10^4$	$3.36 \cdot 10^3$
85 °C Ethylene propylene rubber (EPR)	$2.76 \cdot 10^4$	$1.21 \cdot 10^4$	$3.66 \cdot 10^3$

TABLE VIII
VALUES OF K^2 FOR PROTECTIVE CONDUCTORS AS A CORE INCORPORATED IN A CABLE OR BUNCHED WITH OTHER CABLES OR INSULATED CONDUCTORS

Conductor insulation	k^2 [A²s/mm²]		
	Copper	Aluminum	Steel
Polyvinyl Chloride (PVC) (conductor cross-section ≤ 300 mm²)	$1.32 \cdot 10^4$	$5.79 \cdot 10^3$	$1.75 \cdot 10^3$
Polyvinyl Chloride (PVC) (conductor cross-section > 300 mm²)	$1.05 \cdot 10^4$	$4.63 \cdot 10^3$	$1.40 \cdot 10^3$
Cross-linked Polyethylene (XLPE)	$2.04 \cdot 10^4$	$8.94 \cdot 10^3$	$2.67 \cdot 10^3$
60 °C Ethylene Propylene Rubber (EPR)	$1.98 \cdot 10^4$	$8.69 \cdot 10^3$	$2.62 \cdot 10^3$
85 °C Ethylene propylene rubber (EPR)	$1.79 \cdot 10^4$	$7.87 \cdot 10^3$	$2.36 \cdot 10^3$

TABLE IX
VALUES OF K^2 FOR BARE PROTECTIVE CONDUCTORS IN CONTACT WITH CABLE COVERING, BUT NOT BUNCHED WITH OTHER CABLES

Cable covering	k^2 [A²s/mm²]		
	Copper	Aluminum	Steel
Polyvinyl Chloride (PVC)	$2.53 \cdot 10^4$	$1.11 \cdot 10^4$	$3.36 \cdot 10^3$
Cross-linked Polyethylene (XLPE)	$1.90 \cdot 10^4$	$8.38 \cdot 10^3$	$2.55 \cdot 10^3$
Chlorosulphonated Polyethylene (CSP)	$2.76 \cdot 10^4$	$1.21 \cdot 10$	$3.66 \cdot 10^4$

TABLE X
VALUES OF K^2 FOR PROTECTIVE CONDUCTORS AS A METALLIC LAYER OF A CABLE

Cable insulation	k^2 [A²s/mm²]			
	Copper	Aluminum	Lead	Steel
70°C Polyvinyl chloride (PVC)	$1.98 \cdot 10^4$	$8.69 \cdot 10^3$	$6.67 \cdot 10^2$	$2.62 \cdot 10^4$
90°C Polyvinyl chloride (PVC)	$1.65 \cdot 10^4$	$7.22 \cdot 10^3$	$5.54 \cdot 10^2$	$2.17 \cdot 10^3$
Cross-linked polyethylene (XLPE)	$1.65 \cdot 10^4$	$7.22 \cdot 10^3$	$5.54 \cdot 10^2$	$2.17 \cdot 10^4$
60 °C Ethylene propylene rubber (EPR)	$2.07 \cdot 10^4$	$9.08 \cdot 10^3$	$6.97 \cdot 10^2$	$2.73 \cdot 10^3$
85 °C Ethylene propylene rubber (EPR)	$1.96 \cdot 10^4$	$8.58 \cdot 10^3$	$6.59 \cdot 10^2$	$2.57 \cdot 10^3$
Mineral PVC covered	$1.81 \cdot 10^4$	n/a	n/a	n/a
Mineral bare sheath	$1.81 \cdot 10^4$	n/a	n/a	n/a

TABLE XI
VALUES OF K^2 FOR BARE PE WHERE THERE IS NO RISK OF DAMAGE TO ANY NEIGHBORING MATERIAL

Conditions of the bare PE	k^2 [A²s/mm²]		
	Copper	Aluminum	Steel
Visible and in restricted areas	$5.20 \cdot 10^4$	$1.57 \cdot 10^4$	$6.77 \cdot 10^3$
In normal conditions areas	$2.53 \cdot 10^4$	$1.11 \cdot 10^4$	$3.36 \cdot 10^3$
In fire risk areas	$1.90 \cdot 10^4$	$8.38 \cdot 10^3$	$2.55 \cdot 10^3$

TABLE XII
A COMPARISON OF VALUES OF K^2 OBTAINED WITH DIFFERENT FORMULAE (PROTECTIVE CONDUCTORS NOT INCORPORATED IN CABLES, AND NOT BUNCHED WITH OTHER CABLES—XLPE)

XLPE (θ_M= 250°C)	k^2 (per Eq. (2))	k_{30}^2 (per Eq. (5) and (6))	k_{90}^2 (per Eq. (5) and (6))
1	2	3	4
θ_0	30°C	30°C	90°C
Copper	3.08·10⁴	3.04·10⁴	2.02·10⁴
Aluminum	1.35·10⁴	1.28·10⁴	8.49·10³

IV. VALUES OF K^2 FOR PROTECTIVE CONDUCTORS

Based on initial and limit temperatures listed in Tables I–VI, the k^2 factor can be calculated according to (2) for any given insulation and conductive material of wires. Tables VII–XI report the results of the calculation:

The value of the k^2 factor can also be determined through formulas currently present in literature; however, attention must be paid to such formulas and on the assumptions on which they are based.

In fact, the expression of the k^2 factor in [1] for copper and aluminum are both incorrect. The Power Systems Grounding Working Group of the Technical Books Coordinating Committee IEEE P3003.2 "Recommended Practice for Equipment Grounding and Bonding in Industrial and Commercial Power Systems" is aware of this issue. Thus, in the new dot standard P3003.2, based on [1, Ch. 2], the following formulas, found in [9] and [10] are being proposed for equipment grounding conductors, respectively, in copper and aluminum:

$$\left(\frac{I^2}{A^2}\right) t = 0.0297 \log_{10}\left(\frac{T_M + 234}{T_i + 234}\right) \tag{5}$$

$$\left(\frac{I^2}{A^2}\right) t = 0.0125 \log_{10}\left(\frac{T_M + 228}{T_i + 228}\right) \tag{6}$$

where I is the fault current through the conductor in amperes, A is the protective conductor cross-sectional area in circular mils, t is the fault clearing time in seconds, T_i is the initial operating temperature in degrees Celsius and T_M is the maximum temperature for no damage in degrees Celsius. Reference [1] also indicates that T_i is often taken as the conductor maximum operating temperature in correspondence with its current-carrying capability rather than its initial temperature. This is a conservative approach of which the designer should be aware, as may result in protective conductor oversizing.

A comparison between the values of k^2 obtained with (2) and k_{30}^2 ($T_i = 30$ °C and k_{90}^2 ($T_i = 90$ °C) calculated with (5) and (6) is shown in Table XII for the case of protective conductors not incorporated in cables, and not bunched with other cables insulated in XLPE ($\theta_M = 250$ °C).

It can be seen that (2), (5), and (6) substantially provide the same results for $\theta_0 = 30$ °C (columns 2 and 3 of Table XII). However, if $\theta_0 = 90$ °C is employed in (5) and (6), as implicitly allowed in [1], the values of k_{90}^2 (column 4 of Table XII) are more than 30% lesser than the values k^2 calculated with (2) (column 2 of Table XII). It is important to note that reduced values of k^2 determine larger cross-sectional areas for the PE in correspondence with the same ground-fault current and clearing time of protective devices. This conservative approach does compound with the assumption of adiabatic conditions during ground faults in determining larger PE.

V. MINIMUM CROSS-SECTIONAL AREAS OF PROTECTIVE CONDUCTORS IN ADIABATIC CONDITIONS

The analytical evaluation of the integral of the left-hand side of (1) is rather complex, as the ground-fault current is asymmetrical due to the development of a transient dc component [11], [12]. The method proposed in [11] allows the evaluation of the maximum possible thermal stress to conductors involved in faults by taking into account the worst possible asymmetries of fault currents due to both the making angle and the short-circuit phase angle.

However, [4] and [13] indicate that if protective devices can clear the ground fault within 5 s from its inception, the following simplified formula may be used to determine the minimum and safe cross-sectional area $S(\mathrm{mm}^2)$ of PEs in adiabatic conditions:

$$S \geq \frac{I_G}{k}\sqrt{t_f}. \tag{7}$$

I_G is the r.m.s. value of the prospective ground-fault current circulating through the PE for a fault of negligible impedance, and t_f is the operating time of the protective device in correspondence with the ground-fault current. In reality, when the ground-fault current is not constant, the error caused by the simplification shown in (7) is acceptable provided that either the dc transient component of the ground-fault current quickly expires or protective devices do not clear the ground fault within the first cycle. As per the above simplification, the method used in [11] is not herein used.

It is important to note, though, that the optimum wire size of the PE is not per se a guarantee of electrical safety for persons. In fact, also, terminations, joints, bonding jumpers, etc., included within the ground-fault path must have equal, or greater, thermal capabilities than that of the protective conductor.

If (7) produces nonstandard sizes, protective conductors of a higher standard cross-sectional area must be used. In addition, as anticipated in the previous section, the choice of using the maximum operating temperature as θ_0 may lead to further oversizing.

To better understand this issue, let us consider the Time-Inverse trip curve as a function of the prospective ground-fault current for a 20-A molded case circuit breaker (Fig. 1).

The trip curve of Fig. 1 provides the clearing times t_f of the circuit breaker in correspondence with any given value of the ground-fault current. Such values are listed in Table XIII.

Table XIII also shows the values of calculated and trade sizes of cross-sectional areas (columns 4 and 5) as per (7). The two initial temperatures used in the calculation of k^2 are $\theta_0 = 30$ °C and $\theta_0 = 90$ °C, in the case of a copper protective conductor not incorporated in cables, and not bunched with other cables, insulated in XLPE ($k = 175.55$ and $k_{90} = 141.99$, as per Table XII).

It can be clearly seen that the adoption of the operating temperature of 90 °C results in some cases in protective

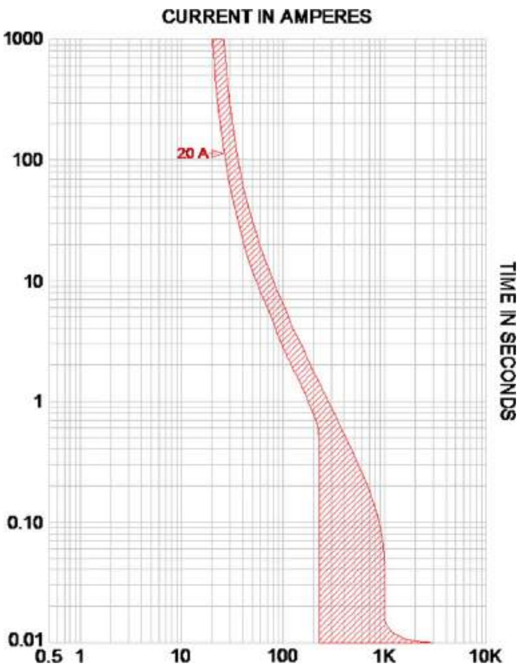

Fig. 1. Time-Inverse trip curve for a 20-A molded case circuit breaker.

TABLE XIII
CALCULATED AND TRADE SIZES OF CROSS-SECTIONAL AREAS OF PROTECTIVE CONDUCTORS NOT INCORPORATED IN CABLES, AND NOT BUNCHED WITH OTHER CABLES—XLPE

I_G(A)	t_f(s)	I_G^2t (A^2s)	$S_{30}\rightarrow S_{30\ trade\ size}$ (mm^2)	$S_{90}\rightarrow S_{90\ trade\ size}$ (mm^2)
1	2	3	4	5
20	991	$3.96\cdot10^5$	3.59→4	4.43→6
30	193	1.74105	2.37→2.5	2.94→4
40	53	$8.48\cdot10^4$	1.66→2.5	2.05→2.5
50	21.6	$5.40\cdot10^4$	1.32→1.5	1.64→2.5
60	20.8	$7.49\cdot10^4$	1.56→2.5	1.93→2.5
100	5.4	$5.40\cdot10^4$	1.32→1.5	1.64→2.5
200	1.73	$6.92\cdot10^4$	1.50→1.5	1.85→2.5
400	0.5	$8.00\cdot10^4$	1.61→2.5	1.99→2.5
500	0.37	$9.25\cdot10^4$	1.73→2.5	2.14→2.5
600	0.23	$8.28\cdot10^4$	1.64→2.5	2.03→2.5
700	0.18	$8.82\cdot10^4$	1.69→2.5	2.09→2.5
800	0.13	$8.32\cdot10^4$	1.64→2.5	2.03→2.5
1000	0.014	$1.40\cdot10^4$	0.67→1.5	0.83→1.5

conductor oversizing by one trade size, in the presence of the same ground-fault current and clearing time.

Equation (7) can of course be used for protective conductors such as armors, metallic sheaths, tapes, etc. In these cases, we consider an equivalent cross-sectional area S_E(mm^2) given by [5]:

$$S_E = \sqrt{\frac{\rho_{20}\cdot P}{R_{20}\cdot\gamma}} \tag{8}$$

where ρ_{20} is the resistivity at 20 °C (Ωmm), R_{20} is the resistance per kilometer at 20 °C (Ωkm^{-1}), P is the mass per kilometer ($kg\ km^{-1}$), γ is the specific mass ($kg\ mm^{-3}$).

TABLE XIV
CONSTANTS X AND Y

Insulation	Copper	
	X $(mm^2/s)^{0.5}$	Y (mm^2/s)
PVC ≤ 3 kV	0.29	0.06
EPR ≤ 3 kV	0.38	0.1
XLPE	0.41	0.12

VI. NONADIABATIC METHOD

The assumption that under ground-fault conditions all the thermal energy is accumulated within the protective conductors may be in some case pessimistic, as heat transfer into the neighboring environment does occur.

References [14] and [15] provide details for the nonadiabatic method, which is valid for all ground-fault durations, and is based on an empirical approach.

If part of the heat is dispersed toward adjacent bodies, the permissible ground-fault current can increase, without risk of damaging the protective conductor of a given cross-sectional area. Alternatively, the wire size of the PE can be safely decreased with respect to the value determined with the adiabatic method.

The permissible nonadiabatic ground-fault current $I_{\rm NAD}$ is given by

$$I_{\rm NAD} = \varepsilon I_{\rm G} \tag{9}$$

where ε is the nonadiabatic factor, which takes into account heat loss into the adjacent components ($\varepsilon = 1$ in adiabatic conditions, whereas ε is > 1 in nonadiabatic conditions); $I_{\rm G}$ is the ground-fault current calculated with (7) in adiabatic conditions.

In the case of insulated conductors as PEs, the nonadiabatic factor is given by the following simplified empirical formula:

$$\varepsilon = \sqrt{1 + XZ + YZ^2}. \tag{10}$$

The constants X and Y for copper protective conductors are presented in the following Table XIV, as a function of the PE insulation, and for voltages ≤3 kV.

Z is defined as

$$Z = \sqrt{t_f/S} \tag{11}$$

where t_f is the ground-fault duration (in s) and S is the PE geometrical cross-sectional area (in mm^2).

For the usual range of wire sizes encountered in the practice, [14] indicates, as a decision-making criterion, to neglect the improvement in the permissible ground-fault current when its increase is less than 5%, that is, when $I_{\rm NAD} < 1.05 I_{\rm G}$. In this case, the nonadiabatic method is not recommended to determine the minimum cross section of PEs.

Based on this criterion, the authors have performed computations based on (10) to determine the values of trade wire sizes for copper conductors, for which $\varepsilon \leq 1.05$. These calculations have taken into account different values of t_f, as well as, trade wire sizes from 1.5 to 300 mm^2. Threshold values for S have been identified, below which the adiabatic hypothesis is too pessimistic and protective conductors result oversized.

TABLE XV
MAXIMUM VALUES FOR S FOR COPPER PEs, FOR WHICH $\varepsilon \geq 1.05$

Insulation	t_f (s)	S (mm^2)	t_f (s)	S (mm^2)	t_f (s)	S (mm^2)	t_f (s)	S (mm^2)
PVC	0.1	n/a	1	10	10	95	100	>300
EPR	0.1	1.5	1	16	10	150	100	>300
XLPE	0.1	1.5	1	16	10	150	100	>300

Table XV lists the smallest values for S for copper wires, for which $\varepsilon \leq 1.05$, for t_f equal to 0.1 s, 1 s, and 10 s.

According to (10), the maximum size for protective conductors for the nonadiabatic condition to be useful in practice is $S = 10$ mm^2, as long as t_f equals at least 0.1 s. Calculations show, in fact, that for fault durations $t_f \leq 0.1$ s, the heat exchange with the surrounding air or materials is negligible (i.e., $\varepsilon \cong 1$) even for the smallest wire trade size of 1.5 mm^2.

VII. CONCLUSION

The authors have proposed an analytical method for the calculation of k^2 for protective conductors, which takes into account the thermal characteristics of insulations, as well as of neighboring materials. This has allowed the determination of the optimum value of S.

The analytical results confirm the formulas currently present in literature and to be adopted in P3003.2 for the adiabatic case. However, such formulas only consider PEs as wires and may lead to oversizing the protective conductors, of which the engineer should be aware.

It has been substantiated, in fact, that two pessimistic choices may be made: using the maximum operating current of protective conductors rather than the ambient temperature; considering the thermal phenomenon developing during ground faults always adiabatic.

Compiling these two assumptions may lead to oversized protective conductors by one or two trade sizes.

APPENDIX

In adiabatic conditions, during the ground fault, the following thermal balance occurs:

$$\rho \cdot \frac{l}{S} \cdot i_G^2 dt = S \cdot l \cdot c \cdot d\theta \tag{A1}$$

where l is the length of the ECG, S its cross-sectional area (mm^2), ρ its resistivity (Ωmm), and c its volumetric heat capacity (J/($^\circ$C mm^3)); i_G is the instantaneous ground-fault current.

The left-hand side of (A1) quantifies the heat developed by the fault current during the infinitesimal time dt, while the right-hand side is the heat accumulated in the conductor during the same time. $d\theta$ is the difference between the initial temperature θ_0 of the conductor, at the inception of the fault, and its temperature θ_f, after the fault is cleared.

The resistivity ρ of the PE is a function of the temperature θ imposed by the ground fault and therefore does not remain constant. In general, $\rho(\theta)$ is not a linear function of the temperature; however, if we assume that the temperature varies in a small range, we can approximate $\rho(\theta)$ with a Taylor series. The Taylor series is a linear representation of a function as an infinite sum of terms based on the values of its derivatives evaluated at an initial point. It is normally acceptable for accuracy to use a finite number of terms of the series to approximate the function.

As a consequence, the resistivity can be written as a Taylor polynomial as a function of θ:

$$\begin{aligned}\rho(\theta) &= \sum_{n=0}^{\infty} \frac{\rho^{(n)}(\theta_0)}{n!}(\theta-\theta_0)^n \cong \rho(\theta_0)+\rho^{(1)}(\theta_0)(\theta-\theta_0)\\ &= \rho(\theta_0)+\rho(\theta_0)\alpha_{\theta_0}(\theta-\theta_0) = \rho(\theta_0)[1+\alpha_{\theta_0}(\theta-\theta_0)]\end{aligned} \tag{A2}$$

where the subscript (n) indicates the nth derivative of the resistivity ρ with respect to the temperature θ, evaluated at the initial temperature θ_0; $\rho(\theta_0)$ represents the resistivity of the PE at the initial temperature θ_0; $\alpha_{\theta 0} = \rho^{(1)}(\theta_0)/\rho(\theta_0)$ is the temperature coefficient of resistivity of the material of the PE at the initial temperature $\theta 0$.

We can write

$$\rho(\theta) = \rho(0^\circ)(1 + \alpha_0 \theta). \tag{A3}$$

To calculate the heat accumulated in the protective conductor during the ground fault, we need to integrate left- and right-hand sides of (A1) between: the instant $t = 0$ of the inception of the fault, and the instant t_f of disconnection of the supply; the temperature θ_0 of the PE at the inception of the fault and its final temperature θ_f when the fault is cleared.

We obtain

$$\int_0^{t_f} i^2 dt = c \cdot S^2 \int_{\theta_0}^{\theta_\mathrm{f}} \frac{d\theta}{\rho} = \frac{c \cdot S^2}{\rho(0^\circ)} \int_{\vartheta_0}^{\vartheta_f} \frac{d\theta}{(1+\alpha_0\theta)}. \tag{A4}$$

Solving the above integral by substitution of the variable θ [3], we obtain

$$\int_0^{t_f} i^2 dt = \frac{cS^2}{\alpha_0\rho_0} \int_{1+\alpha_0\cdot\theta_0}^{1+\alpha_0\cdot\theta_f} \frac{dx}{x} = \frac{cS^2}{\alpha_0\rho_0} \ln\frac{1+\alpha_0\theta_f}{1+\alpha_0\theta_0}. \tag{A5}$$

In order to prevent damages to the insulation of the PE, the final temperature θ_f in (A5) must not exceed the maximum temperature θ_M that its insulation can withstand. Hence, if we replace θ_f with θ_M, we can define the parameter k^2 as

$$k^2 = \frac{c}{\alpha_0 \cdot \rho_0} \ln\frac{1+\alpha_0\theta_M}{1+\theta_0\alpha_0}. \tag{A6}$$

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